Persistent Hall Voltage and Current in the Fractional Quantum Hall Regime

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The persistent Hall voltage and current in an isolated annulus in a strong perpendicular magnetic field, at filling factor $\nu = 1/q$, and in the presence of a weak constriction is obtained as a function of temperature, and flux piercing the annulus. A thermodynamic Hall conductance is found which has a universal value even with back scattering at the constriction.

I. INTRODUCTION

The trial wavefunction proposed by Laughlin was established to describe very well the ground state of a two-dimensional electron gas in a strong perpendicular magnetic field at filling factors $\nu = 1/m$, m odd as an incompressible state [1]. However, it is not yet clear if this implies that the excitations of this ground state have fractional charge and fractional statistics.

In this article we consider a 2- dimensional annulus in a strong magnetic field which condenses the electrons to a fractional quantum hall state. We will call this a FQH-annulus in the following. The Laughlin wave function for a pure FQH- annulus when the filling factor satisfies $\nu = 1/m$, where m is an odd integer, is given by [2]:

$$\psi = \prod_{i=1}^{N} \mid z_i \mid^{\varphi} z_i^n \exp(-\frac{\mid z_i \mid^2}{4 l_B^2}) \prod_{j=1}^{i-1} (z_i - z_j)^{1/\nu}, \qquad (1)$$

where $n + \varphi$ is the total flux piercing the FQH- annulus. The inner and outer radii of the incompressible FQH liquid in the annulus are given by $r^2 = 2 l_B^2 (n + \varphi)$ and $R^2 = 2 l_B^2 (n + N + \varphi)$, respectively, N being the number of fluxquanta in the area of the annulus. Increasing the flux ϕ through the annulus one thereby increases the radii of the ring. Since the annulus is confined by a certain edge potential, this decreases the energy of the highest filled state at the inner edge while the one at the outer edge is increased. Increasing the flux further, until one flux quantum $\phi_0 = hc/e$ more pierces the annulus, the ground state of the FQH- annulus is identical to the one without flux. However, in order to relax to this ground state, equilibration between the edges of the FQH- annulus is necessary.

The thermal equilibration can be due to backscattering across the bulk, since we consider an isolated FQH- annulus which is not connected to any leads. It was predicted by Thouless and Gefen, that thermodynamic properties of such a system have flux periodicity ϕ_0 due to the existence of families of states of the FQH- annulus, which are connected by a physical mechanism, the finite back scattering amplitude of fractionally charged quasiparticles. If only electrons could backscatter from edge to edge, the flux periodicity would be enhanced, in violation of the Byers and Young theorem [3]. Gefen and Thouless argued that this could be a proof of the existence of fractional charge excitations.

Thouless and Gefen considered in Ref. [2] a nonequilibrium situation in the presence of leads, with a time dependent flux through the annulus. Here, rather we want to restrict us to the thermal equilibrium of an isolated annulus in the presence of a weak constriction, see Fig. I. The other limit of a strong constriction, the weak link limit of the FQH- annulus is considered in a separate article [4].

We will use a model of the FQH- annulus to establish not only the periodicity of the persistent current, but also its magnitude. The Fractional Quantum Hall liquid has gapless excitations at its edges while the bulk excitations have a gap [9]. In this article we restrict ourselves to the case of a sharp edge potential. Then, the FQH-ring has two branches of edge excitations, one at the inner and one at the outer edge. Special care has to be taken of the zero modes of the edge excitations, which describe the removal and adding of particles.

Additionally, we introduce a local back scattering impurity, which models the scattering of particles between the inner and outer edge. Since it has been shown that such a back scattering amplitude in a FQH bar is strongly renormalized and thus effectively temperature dependent [6,8], this will be an additional source of the temperature dependence of the persistent current.

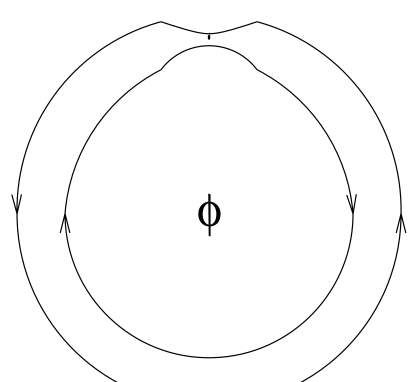

FIG. 1. An annulus of a 2- dim. Electron- gas with a weak constriction, in a strong magnetic field, the arrows denote the chirality of the edge states

The back scattering amplitude of fractionally charged quasiparticles is found to be enhanced as the temperature is lowered, so that one could even expect that the persistent current is depressed as the temperature is lowered. This unusual expectation is another motivation for us to derive the persistent current in a FQH- annulus with a weak impurity as a function of temperature in the following.

In the next section we introduce the edge state theory for the special geometry of a FQH- annulus. Then, we present the derivation of the persistent current and hall voltage in an isolated FQH- annulus, and discuss the result obtained.

The more technical derivations are included in the appendices. In appendix A, the hamiltonian of the gapless edge excitations is derived in a hydrodynamic model [9]. In appendix B, the quantization of the gapless edge excitations and the bosonization of the quasiparticle operators is given. In appendix C we present the path integral derivation of the functional integral representation of the partition function. In appendix D, the renormalization of the back scattering amplitude is derived. In the last appendix E the solution of series and integrals occuring in this article are listed.

II. THE FQH- ANNULUS WITH A CONSTRICTION

The hamiltonian of the gapless edge excitations at the edge of the FQH- annulus at filling factor $\nu = 1/q$, q odd, is given by (see appendix A)

$$H_0 = \frac{\pi}{\nu} \sum_{p=\pm} \int_0^{L_p} dx v_p (\rho_p(x) + p\nu \frac{\varphi}{2\pi R_p})^2. \qquad (2)$$

Here, $\rho_p(x)$ is the chiral electron density along the edge, $L_p = 2\pi R_p$ is the length of each edge.

Note that the hamiltonian depends on the flux φ piercing the annulus additionally to the homogenous constant magnetic field B. This accounts for the fact that as the flux φ is varied, the eigenstates are pushed up the outer edge and down the inner edge as first noted by Halperin [10], thus changing the total energy of the edge states. Indeed, we see in Eq. (2), that the energy of the outer edge $p = +$, increases when increasing the flux $\varphi = \phi/\phi_0$, while the energy of the inner edge, $p = -$ decreases.

We consider the weak back scattering case by an additional term in the Hamiltonian due to the short range backscattering impurity given by

$$H_{BS} = \sum_{m=1} t_m \sum_{l=0}^{1/\nu-1} (\psi_+^{+m}(lL_+)\psi_-^m(lL_-) + c.c.). \qquad (3)$$

Here, $m = 1$ corresponds to backscattering of quasiparticles with charge ν, while $m = 1/\nu$ describes the backscattering of chiral electrons. The summation over l takes account of the fact that the quasiparticle operator $\psi(x)$ is $1/\nu L_p$- periodic. Thus, the quasiparticle tunneling between the points $x = 0, L_p, ..., (1/\nu - 1)L_p$ are distinguishable, so that the sum over them is explicit.

Next, the Hamiltonian can be quantized and the quasiparticle operators $\psi_p(x)$ can be bosonized in terms of the edge magnetoplasmon modes. We give the details in appendix B. With this microscopic theory of edge excitations, we are now able not only to obtain the periodicity of thermodynamic properties, but also the magnitude of the persistent hall voltage and current in the annulus as a function of the flux piercing it, its circumference and the temperature.

III. THE PERSISTENT HALL VOLTAGE AND CURRENT IN A FQH- ANNULUS

Having the full Hamiltonian and the complete algebra, we can derive the partition function of the FQH- annulus at finite temperature T. By means of the path integral method the partition function can be expressed through a functional integral, see appendix C. One obtains,

$$Z = \int \prod_{\tau=0}^{\beta} \prod_{x=-L_p/2}^{L_p/2} \prod_{p=\pm} d\phi_p(x) \exp[-S] \mid_{N=const.} . \qquad (4)$$

Here, the action S is given by (see Eq. (C14)),

$$\begin{aligned} S = & \int_0^\beta d\tau \sum_{p=\pm} \int_{-L_p/2}^{L_p/2} dx (p \frac{i}{4\pi\nu} (\partial_\tau \partial_x \phi_p(x))\phi_p(x) \\ & + \frac{\pi v_p}{\nu} (p \frac{1}{2\pi} \partial_x \phi_p(x) + p\nu \frac{\varphi}{2\pi R_p})^2) \\ & + \int_0^\beta d\tau \sum_{m=1} t_m 2 \sum_{l=0}^{1/\nu-1} \cos m[\phi_+(lL_+) - \phi_-(lL_-) + \frac{\pi\nu}{2} N] \\ & - \int_0^\beta d\tau \sum_{p=\pm} p \frac{i}{4\pi\nu} \phi_p(x) \partial_\tau \phi_p(-x) \mid_0^{L_p/2} . \end{aligned} \qquad (5)$$

Note the last, new term which is due to the selfduality of the chiral edge field operators, and which is nonvanishing when the zero- modes are dynamic. See appendix C for details. We introduced $N = N_+ + N_-$ as the total quasiparticle number on the inner and outer edge of the FQH- annulus. The additional phase factor in the back scattering hamiltonian is due to the anticommutation between chiral fermions on the inner and the outer edge. Since $N_p = p\phi_p(L_p/2) - p\phi_p(-L_p/2)$, the path integral derivation could be performed as outlined in appendix C even in the presence of these statistical factors C_p in the bosonized quasiparticle operators, Eqs. (B2, B20).

In order to perform the functional integrals, it is convenient to represent it in the momentum representation,

Eq. (B4), where we take the basis of magnetoplasmon modes of the annulus without impurity:

$$Z = \int \prod_{\tau=0}^{\beta} dJ(\tau) d\phi_J \prod_{p=\pm} \prod_{n\neq 0} d\phi_{p,n}(\tau) \exp[-S] \mid_{N=const.} . \tag{6}$$

Here, we introduced the notation $J = N_+ - N_-$ with its conjugate, the phase difference between the two edges, $\phi_J = \phi_{+0} - \phi_{-0}$. The action in momentum space is:

$$S = \int_0^\beta d\tau (\sum_{p=\pm} (\frac{i}{2}\phi_J \partial_\tau J - \sum_{n>0} \frac{n}{\nu}(\partial_\tau \phi_{p,n})\phi_{+,-n}) + H[\phi_+, \phi_-]), \tag{7}$$

where

$$H[\phi_p] = \sum_{p=\pm} H_p[\phi_p] + H_{BS}[\phi_+, \phi_-]. \tag{8}$$

The flux- dependent potential energy of edge p is diagonal in momentum,

$$H_p[\phi_p] = \frac{\nu v_p}{2R_p}((N + pJ)/2 + p\varphi)^2 + \frac{1}{\nu}\frac{v_p}{R_p}\sum_{n>0} n^2 \phi_{p,n}\phi_{p,-n}, \tag{9}$$

while the back scattering term of the Hamiltonian mixes the momentum Eigenstates of the clean FQH- annulus,

$$H_{BS}[\phi_p] = \sum_{m=1}^{\infty} 2t_m \sum_{l=0}^{1/\nu-1} \cos m[\phi_J + 2\pi\nu l N + \sum_{n\neq 0}(\phi_{p,n} - \phi_{-p,n}) + \frac{\pi}{2}\nu p N)]. \tag{10}$$

Note that the action does not depend on $\phi_N = \phi_1 + \phi_2$, but, in the tunneling term, on the phase difference ϕ_J, since the total particle number N is kept fixed, $\partial_\tau N = 0$, in the isolated FQH- annulus.

We make at this point an interesting observation: When there is a fractional total charge N, the back scattering term of m quasiparticles is exactly zero due to the sum over l, for all m besides $m = 1/\nu n$, n integer, corresponding to electron back scattering.

Restricting N to multiples of $1/\nu$ in the following, which makes sense since the total charge of the FQH- annulus should not be fractional, we obtain thus:

$$H_{BS}[\phi_p] = \sum_{m=1}^{\infty} 2t_m 1/\nu \cos m[\phi_J + \sum_{n\neq 0}(\phi_{p,n} - \phi_{-p,n}) + \frac{\pi}{2}\nu p N)]. \tag{11}$$

For simplicity, we substitute $2/\nu t_m$ by t_m in the following.

Now, we already can conclude from the action, Eq. (7), that the flux- periodicity of the partition function is ϕ_0. To this end, we note that the only flux dependent term of the action is the potential energy Eq. (9). Thus, a change of φ by 1 does not change the partition function if there is a sum over families of different numbers of fractionally charged quasiparticles N_p, as noted by Thouless and Gefen [2], and which is explicit in the functional integral representation given here.

Next, we study how the back scattering amplitudes t_m are renormalized as a function of a typical energy, temperature or level spacing, $E = max(T, v_p/R_p)$, by perturbative renormalization theory. One finds as outlined in appendix D:

$$t_{m,eff}(E) = t_m(E/\Lambda)^{m^2\nu-1} e^{-\nu m^2 v/L/E/2}. \tag{12}$$

where $v/L = (v_+/L_+ + v_-/L_-)/2$. Thus, for temperatures much exceeding the level spacing v/R, the quasiparticle backscattering amplitude, m=1, is enhanced when the temperature is lowered like a power law in temperature, as found in Ref. [6,8], while higher order backscattering amplitudes, $m^2 > 1/\nu$ are irrelevant. However, we additionally find an exponential prefactor, which becomes dominant, when the temperature approaches the finite level spacing v/R. This is reminiscent of the Coulomb blockade effect, where the charging energy is here the level spacing, the energy of an additional particle on the edge. When the temperature is lowered below the level spacing, the renormalized tunneling amplitude saturates at a temperature independent value, which increases like a power law, when the level spacing is reduced. This is quite surprising, since intuitively, one should think that the effect of a single back scattering impurity rather decreases, when the extension of the edge states is increased.

Before proceeding in the calculation of the functional integrals, we consider first the expressions for the persistent hall voltage and the persistent current. The thermodynamic persistent current is given by the flux derivative of the free energy:

$$I(\varphi) = T\frac{e}{2\pi}\partial_\varphi \ln Z. \tag{13}$$

Using the above functional integral representation of the partition function Z, we see that only the potential energy is flux dependent and we obtain:

$$I(\varphi) = -\nu\frac{e}{2\pi}\frac{v}{R} < T\int_0^\beta d\tau(J(\tau) + 2\varphi) > - \nu\frac{e}{4\pi}(\frac{v_+}{R_+} - \frac{v_-}{R_-})N. \tag{14}$$

where

$$< ... >= \int \prod_{\tau=0}^{\beta} dJ d\phi_J \prod_p \prod_{n\neq 0} \phi_{pn}[...] \exp[-S]/Z. \tag{15}$$

We see that there is even a finite persistent current without external flux due to the difference in energy for a particle added to the inner or the outer edge.

Next, we define a Hall voltage by noting that the voltage at a given imaginary time τ is related to the imaginary time derivative of the phase difference ϕ_J:

$$V(\tau) =< 1/(\nu e)\partial_\tau \phi_J >= -1/(2\nu e) < \partial_J H(\tau) > . \quad (16)$$

This is a quantity averaged over the circumference of the ring. We note that that there are additionally local fluctuations of the edge to edge voltage. Averaging over the imaginary time τ, we obtain the thermodynamic Hall voltage. Then, we find that this persistent Hall voltage is related to the persistent current by a thermodynamic Hall conductance

$$\sigma_{xy} = \frac{I}{V} = \nu \frac{e^2}{2\pi\hbar}. \quad (17)$$

It is noteworthy that this thermodynamic hall conductance is universal even in the presence of back scattering between the edges. This is in contrast to the transport Hall conductance which defers from this universal value as soon as there is back scattering, that is when there is no transversal localization between the edges.

The reason for this drastically different behaviour of the thermodynamic Hall conductance is clear: In the transport experiment a current is driven through the sample externally and the quantum hall bar can only dynamically react to this external perturbation. Back scattering between the edges reduces the Hall voltage but can not affect the externally driven current, enhancing thus the Hall conductance. In the isolated FQH- annulus on the other hand backscattering not only reduces the Hall voltage but also the thermodynamic current, so that the Hall conductance is unchanged.

Now, we proceed in the explicit calculation of the persistent current in the presence of the single backscattering impurity. We do an expansion in the scattering amplitudes t_m, in order to be able to perform the functional integrals explicitly.

The partition function has after expansion in the tunneling amplitudes t_m the form:

$$Z = \sum_{w=0}^{\infty} \frac{1}{w!} \sum_{m_1 ... m_w = 1}^{\infty} \prod_{l=1}^{w} t_{m_l} \int_0^\beta d\tau_1 ... d\tau_w$$
$$\int \prod_{\tau=0}^{\beta} dJ(\tau) d\phi_J(\tau) \prod_p \prod_{n\neq 0} d\phi_{p,n}(\tau) \sum_{\alpha_1,...,\alpha_w=\pm}$$
$$\exp(-S \mid_{t_m=0}) \exp[i \sum_{l=1}^{w} \alpha_l m_l (\phi_J(\tau_l) + p\frac{\pi}{2}\nu N$$
$$+ \sum_{n\neq 0} (\phi_{+n}(\tau_l) - \phi_{-n}(\tau_l)))] \mid_{N=const.} . \quad (18)$$

The integral over nonzero modes can be performed by going to Matsubara frequency representation (see appendix D), and gives a factor for the w-th order term which is independent on flux φ:

$$\exp(-\sum_{l,l'=1}^{w} \alpha_l \alpha_{l'} m_l m_{l'} D(\tau_l - \tau_{l'})), \quad (19)$$

where one obtains after summation over the Matsubara frequencies,

$$D(\tau) + D(-\tau) = \nu \sum_p \sum_{n>0} \frac{1}{n} \frac{\cosh(n v_p/(2R_p T)(1 - T \mid \tau \mid))}{\sinh(n v_p/(2 R_p T))}. \quad (20)$$

Since the edge magnetoplasmons were considered in harmonic approximation, it is not surprising that this factor due to the gapless nonzero edge modes resembles the Debye- Waller- factor, describing the thermal damping of periodic structures.

Next we can do the remaining integrals over 0-modes:

$$Z_w^0 = \int \prod_{\tau=0}^{\beta} dJ(\tau) d\phi_J(\tau)$$
$$\exp[-\int_0^\beta d\tau (\frac{i}{2}\phi_J \partial_\tau J + \frac{\nu v}{R}(\frac{1}{2}J + \varphi)^2 + \frac{\nu v}{4R} N^2$$
$$- i \sum_{l=1}^{w} \alpha_l m_l (\phi_J(\tau_l) + \pi/2\nu p N))]. \quad (21)$$

Performing first the integral over the phase difference ϕ_J gives a delta- function, which ensures that the change of the particle number difference J on one edge happens only at the tunneling events τ_l: $\partial_\tau J = 2\sum_{l=1}^{w} \alpha_l m_l \delta(\tau_l - \tau)$. Thus, the difference in particle number betweeen the two edges is constrained to be

$$J(\tau) = \bar{J} + 2 \sum_{l=1}^{w} \alpha_l m_l \theta_{step}(\tau - \tau_l), \quad (22)$$

where $\bar{J}$ is an even integer. Note that the periodic boundary conditions $J(\tau + \beta) = J(\tau)$ require that

$$\sum_{l=1}^{w} \alpha_l m_l = 0, \quad (23)$$

exactly, which means that the total change in charge on either edge, after w tunneling events, has to be zero. This is a consequence of the fact that the partition function is a trace, that is, one must come back to the same state when going in the path integral from $\tau = 0$ to $\tau = \beta$. Note that while this condition makes the first order term vanish this is not true for odd terms in general. For example in the third order term, $w = 3$, the condition Eq. (23) can be fullfilled with f.e. $m_1 = m_2 = 1, m_3 = 2, \alpha_1 = \alpha_2 = -\alpha_3$, which corresponds to twice a $2k_F$ scatttering from edge $p = +$ to $p = -$ followed by a $4k_F$ scattering back.

Now, also the integral over J is performed, up to a summation over the integer numbers $\bar{J}$. This is a summation and not a continous integral, since only multiples of the charge of a quasiparticle can be added or removed from one edge, due to the edge to edge back scattering which does not allow a continous transfer of charge. In fact, the persistent current would vanish exactly, if there would be a continous integral rather than a discrete sum over $\bar{J}$. Thus, for the existence of a persistent current it seems essential that only a discrete amount of charge can tunnel between the edges.

Note that at this point the factor

$$\exp(-i\frac{\pi}{2}\nu p\sum_{l=1}^{w}\alpha_l m_l N), \tag{24}$$

originating from the anticommutation between the chiral fermions on the inner and the outer edge dropped out because of the condition Eq. (23), so that we can conclude that it is not essential for the partition function, if the anticommutation between the edges is taken into account or not.

We obtain for the partition function with the condition of neutrality Eq. (23) and transforming to $x_l = T\tau_l$:

$$\begin{aligned} Z = &\sum_{\bar{J}even}\exp(-\frac{\nu v}{4TR}(\bar{J}+2\varphi)^2) \\ &\sum_{w=0}^{\infty}\sum_{m_1,\ldots,m_w}\sum_{\alpha_1,\ldots,\alpha_w=\pm}|_{Eq.23}\prod_{l=1}^{w}(\frac{t_{m_l}}{T}\exp(-m_l^2 D(0))) \\ &\int_0^1 dx_1\int_0^{x_1}dx_2 \ldots dx_w\exp(-\sum_{l\neq l'}\alpha_l\alpha_{l'}m_l m_{l'}D(x_l-x_{l'})) \\ &\exp(\sum_{l=1}^{w}\alpha_l m_l x_l\frac{\nu v}{TR}(\bar{J}+2\varphi-\sum_{l'<l}\alpha_{l'}m_{l'})) \end{aligned} \tag{25}$$

While similar integrals could be performed exactly previously, (see Ref. [11]), this is complicated here

1. by the exponential factor originating from the dynamics of the 0-modes which is not present in the macroscopic limit $R\rightarrow\infty$.

2. the fact that there are relevant mixed terms like f.e. $w=3, m_1=m_2=1, m_3=2, \alpha_1=\alpha_2=-\alpha_3$.

Therefore, we had to give up looking for an exact solution at this point and restrict us now to the second order correction $w=2$. In fact, as we found above by perturbative renormalization theory, Eq. (12), in a finite system the quasiparticle back scattering amplitude saturates quickly as the temperature is lowered, even for temperatures larger then the level spacing $T > v/R$ due to a blockade effect. Thus, we may hope to catch the essential physics of the weak backscattering limit $t_m/\Lambda \ll 1$ by restricting us now to 2 nd order perturbation theory.

The persistent current in the annulus to second order in the back scattering amplitudes t_m is obtained to be given by,

$$\begin{aligned} I = &-\nu\frac{e}{2\pi}\frac{v}{R}(<\bar{J}>_{\bar{J}}+2\varphi+\sum_{m=1}^{\infty}\tilde{t}_m^2 \\ &(<\bar{J}P_m(\bar{J}+2\varphi)>_{\bar{J}}-<\bar{J}>_{\bar{J}}<P_m(\bar{J}+2\varphi)>_{\bar{J}})) \\ &+\frac{e}{2\pi}T\sum_{m=1}^{\infty}\tilde{t}_m^2<\partial_{\varphi}P(\bar{J}+2\varphi)>_{\bar{J}} \\ &-\nu\frac{e}{4\pi}(\frac{v_+}{R_+}-\frac{v_-}{R_-})N, \end{aligned} \tag{26}$$

where we introduced the function

$$\begin{aligned} P_m(z) = &\int_0^1 dx e^{m^2(D(x)+D(-x))} \\ &2\cosh(\frac{\nu v}{TR}mzx)\exp(-\frac{\nu v}{TR}m^2 x). \end{aligned} \tag{27}$$

and the renormalized tunneling amplitude

$$\tilde{t}_m = \frac{t_m}{T}e^{-m^2 D(0)}. \tag{28}$$

The average is taken with respect to the unperturbed system:

$$<\ldots>_{\bar{J}} = \sum_{\bar{J}even}\exp(-\frac{\nu v}{4TR}(\bar{J}+2\varphi)^2)[\ldots]/Z^{(0)}. \tag{29}$$

where $Z^{(0)}$ is the partition function of the unperturbed system.

We see that the persistent current is exponentially small when the temperature T exceeds the level spacing v/R. Therefore, the power law increase with temperature of the back scattering amplitude in this temperature regime has no effect on the magnitude of the persistent current. We see this more clearly by considering the first harmonic of the current, which is for $T > v/R$ given by, keeping only relevant terms,

$$I_1(T) = -2eT\exp(-\frac{\pi^2}{\nu}\frac{T}{v/R})(1-t_1^2(\frac{\pi}{\Lambda})^{2\nu}T^{2(\nu-1)}B(T)). \tag{30}$$

where

$$B(T) = \int_0^1 dx\frac{(\sin\pi x)^2}{(2\sin\pi/2x)^{2\nu}}\exp(\frac{\nu v}{RT}x^2). \tag{31}$$

Thus, the exponential prefactor is dominating down to temperatures T equal to the level spacing, and a decrease in the persistent current when lowering the temperature can not be observed.

When the temperature is below the level spacing, the magnitude of the persistent current increases and higher harmnics become important, so that the shape as a function of flux changes from a sinusoidal to a sawtooth function.

In that limit,

$$\tilde{t}_m^2 P_m(z) = \frac{v/R}{T} t_m^2 (2\Lambda)^{-2m^2\nu} (\frac{v}{R})^{2(\nu m^2-1)} B_m(z, \frac{v/R}{T}), \tag{32}$$

where

$$B_m(z, \frac{v/R}{T}) = \int_0^{v/R/T} dy \frac{2\cosh(\nu m z y)}{(2\sinh(y/4))^{2\nu}} \exp(-\frac{\nu}{2} y). \tag{33}$$

Now, we consider the flux interval $-1/2 < \varphi < 1/2$, and the result is understood to be extended periodically. For $T \ll v/R$ and $|\ \varphi\ | \ll 1/2$, the current is well approximated by using $< \bar{J} >= 0$. Then, keeping relevant terms, only, one obtains,

$$I(\varphi) = -\nu \frac{e}{\pi} \frac{v}{R} \varphi + \nu \frac{e}{2\pi} (\frac{v}{R})^{2\nu-1} t_1^2 (2\Lambda)^{-2\nu} \partial_\varphi B_1(\varphi, \infty). \tag{34}$$

Therefore, we can conclude that the persistent current is temperature independent for temperatures far below the level spacing, and not too close to the turning points $|\ \varphi\ |= 1/2$. Furthermore, the amplitude of the back scattering correction increases like a power law with decreasing level spacing, $\sim (v/R)^{2\nu-1}$, thus reducing the persistent current.

In Fig. 2 we plot the persistent current, Eq. (26) for several temperatures below the level spacing and compare it with the one obtained for the FQH- annulus without constriction, plotted in Fig. 3. We observe that only at very low temperatures a hundred times smaller than the level spacing the reduction due to back scattering becomes appreciable. The peculiar behavior of the current for $T/(v/R) = 1/100$ and flux close to $\pm 1/2$, indicates a breakdown of second order perturbation theory in this regime.

In summary, the persistent current in a FQH- annulus with a weak backscattering impurity was calculated. It was proven that the persistent current is flux periodic with period ϕ_0 due to the possibility of fractionally charged quasiparticles to scatter between the edges of the FQH- annulus. At temperatures T exceeding the level spacing v/R, the current is exponentially small, and the back scattering correction, which increase like a power law, when reducing the temperature, has no effect on the magnitude of the current.

At temperatures far below the level spacing, the current becomes temperature independent, and the back scattering correction increases like a power law, when the level spacing is reduced.

The persistent current is found to be proportional to the fractional charge νe of the quasiparticles, see Eq. (26).

We also found that one can define a thermodynamic Hall conductance which has the universal value $\sigma_{xy} = \nu e^2/(2\pi\hbar)$, even in the presence of back scattering between the edges.

Experimentally, the total magnetization of a FQH- annulus could be measured, where a combination of low temperatures and small samples is preferable in order to observe a signal due to the persistent current. The magnitude of the Hall voltage calculated above is presumably only accesible experimentally, if it is combined with a simultanous measurement of the magnetization, thus making it possible to subtract externally produced voltage fluctuations which cause Hall current fluctuations in the ring.

An interesting question is, if the persistent current changes its magnitude, when the filling factor is changed away from $\nu = 1/q$, or remains on a plateau, reminiscent of the quantum Hall effect [12,13]. Since the method employed here relies only on the incompressibilty of the bulk of the FQH- annulus we expect the result to hold as long as there is a gap for bulk excitations. At temperatures below the level spacing v/R, there is a dependence on magnetic field due to the relation to the edge velocity: $v = E/B$, where E is the gradient of the edge potential. Additionally there might be a strong hysteresis effect, since when changing the total magnetic field it might be impossible to fix the additional flux φ through the ring, making it hard to find the functional dependence on the magnetic field, Eq. 26.

IV. ACKNOWLEDGEMENT

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APPENDIX A: THE HAMILTONIAN OF THE GAPLESS EDGE EXCITATIONS

The effective Hamiltonian of the gapless edge excitations can be found as the gapless excitations of the two-dimensional Chern- Simons- theory describing the interacting electrons in the strong magnetic field in the 2-dimensional annulus [9,14]. Instead, we give here a hydrodynamic derivation of the Hamiltonian of gapless edge excitations, using only the incompressibilty of the bulk system and the existence of a sharp edge [9].

This derivation is valid provided that

i) the edge is sharp, that is the thickness over which the potential varies by the amount of the cyclotron frequency $\omega_c = eB/m$ is smaller than the magnetic length.

ii) the long range interaction is small enough (the gapless edge excitations have a dispersion $E_k = v(k)k$ where $v(k)$ is constant for $k > 1/l_B \exp(-\pi/\nu\omega_c/(e^2/\epsilon/l_B))$. where l_B is the magnetic length, given by $l_B^2 = c/e/B$, the dielectrical constant is typically of the order of $\epsilon \approx 10$, and ω_c the cyclotron frequency [14]. Additionally, interedge interactions are disregarded in the following [15].

Now, we outline the hydrodynamic derivation. Consider a quantum hall bar having an edge potential which is for small deviations y_p from the edges linear: $E_p y_p$. Then we can derive an effectively one- dimensional Hamiltonian by measuring the energy of deviations in the shape of the incompressible bulk at the edge. We will measure those deviations normal to the surface of the quantum hall bar. Thus, y_- is positive if the FQH-liquid rises beyond inner edge, while y_+ is positive if the FQH- liquid moves up the outer edge. This way, we find for the energy of the quantum hall bar due to deviations from the edge of minimal energy:

$$H_0 = \sum_{p=\pm} \int_{edgep} ds \int_0^{h_p(s)} dy_p enE_p y_p. \tag{A1}$$

Note, that the magnetic field perpendicular to the annulus causes a drift velocity along the edge given by $v_p = \nu eE_p/2\pi n$ where $\nu = \phi_0 n/B$ is the filling factor, the amount of particles per flux quantum $\phi_0 = hc/e$, with n being the particle density. Introducing $\rho_p(s) = nh_p(s)$, where $p = \pm$, which have the dimensions of 1-dimensional densities and can be viewed as the particle densities along the edges, we obtain in harmonic approximation:

$$H_0 = \frac{\pi}{\nu} \sum_{p=\pm} v_p \int_{edgep} ds \rho_p(s)^2.. \tag{A2}$$

APPENDIX B: QUANTIZATION

In this appendix we quantize the Hamiltonian of the gapless edge excitations, and obtain the algebra of the edge excitations. The system is characterized by a continuity equation for the density of chiral electrons on each edge of length L_p.

$$\left(\partial_t + pv_p\partial_x\right)\rho_p(x,t) = U_p(x,t), \tag{B1}$$

where $p = \pm$ enumerates the edges and the source and drain function $U_p(t)$ is due to the possibility of back scattering from edge to edge across the bulk at an impurity.

The quasiparticle operators $\psi_p(x)$, removing a quasiparticle of charge νe from edge $p = \pm$, can be related to the chiral electron density at the edge, $\rho_p(x)$ by the bosonization technique.

$$\psi_p(x) = C_p \exp(i\phi_p(x)), \tag{B2}$$

where

$$\rho_p(x) = p\frac{1}{2\pi}\partial_x\phi_p(x), \tag{B3}$$

with $p = \pm$ denoting the edges of opposite chirality. C_p is an operator ensuring anticommutation between chiral electron operators on opposite edges, and will be specified below. Since $\rho_p(x)$ is the electron density along the edge p, $\phi_+(x) = 2\pi\int^x dx'\rho_+(x')$ counts 2π times the number of chiral electrons at positions on the right going edge, $+$, smaller than x, while $\phi_-(x) = 2\pi\int_x dx'\rho_-(x')$ counts accordingly 2π times the number of particles on positions $x' > x$ on the left going edge. Here, the difference in the definition for the two edges arises from the fact that we have chosen the same coordinate system for both edges, whose chirality is opposite, however.

Besides the linear dependence on x due to a finite number of quasiparticles on an edge, N_p, we can use a Fourier expansion for $\phi_p(x)$ and get:

$$\phi_p(x) = \phi_{p0} + p\nu N_p x/R_p + \sum_{n\neq 0}\phi_{pn}e^{inx/R_p}. \tag{B4}$$

The density of chiral electrons is taken to be periodic: $\rho_p(x+L_p) = \rho_p(x)$, and its Fourier expansion is

$$\rho_p(x) = \nu\frac{N_p}{2\pi R_p} + \sum_{n\neq 0}\rho_p(n)e^{inx/R_p}. \tag{B5}$$

The Fourier components of the chiral electron density $\rho_p(n)$ are related to the Fourier components of the phase operator $\phi_p(n)$ as

$$\rho_p(n) = ip\frac{n}{2\pi R_p}\phi_p(n) \tag{B6}$$

for $n \neq 0$.

Quantization of the nonzero modes

Now, from the classical Hamiltonian equations combined with the continuity equation Eq. (B1), we can obtain after quantization the algebra for the density operators in momentum space.

We first consider the Hamiltonian equations which define for the Fourier components of the field, $\phi_{p,n}$, their conjugate fields $P_{p,n}$:

$$\partial_t P_{p,n} = -\frac{\partial H}{\partial\phi_{p,n}}. \tag{B7}$$

The Hamiltonian Eq. (2) of the edge excitations is in momentum representation, using Eqs. (B6, B5) , found to be given by

$$H_0 = \sum_{p=\pm}\frac{\nu v_p}{2R_p}(N_p + p\varphi)^2 + \frac{1}{\nu}\sum_{p=\pm}\frac{v_p}{R_p}\sum_{n>0}n^2\phi_{p,n}\phi_{p,-n}. \tag{B8}$$

Thus, we find that the Hamiltonian equation becomes

$$\partial_t P_{p,n} = -\frac{2\pi v_p}{\nu} n^2 \phi_{p,-n}(t) - \frac{\partial H_{BS}}{\partial \phi_{p,n}}. \quad \text{(B9)}$$

Here, H_{BS} is the part of the Hamiltonian which describes backscattering of quasiparticles from an impurity. Combining the continuity equation Eq. (B1) with this Hamiltonian equation, we thus can identify the conjugate field as

$$P_{p,n} = ip\frac{n}{\nu}\phi_{p,-n}(t), \quad \text{(B10)}$$

provided that the source and drain term in the continuity equation is related to the impurity Hamiltonian as

$$U_{p,-n}(t) = -\frac{\nu}{2\pi R_p}\frac{\partial}{\partial \phi_{p,n}} H_{BS}. \quad \text{(B11)}$$

Having obtained a complete set of conjugate fields, $\phi_{p,n}, P_{p,n}$, for $n > 0$, we can quantize the thus identified conjugate fields and get

$$[p_{p,n}, \phi_{p',n'}] = i\delta_{nn'}\delta_{p,p'}, \quad \text{(B12)}$$

from which follows for the Fourier components of the density operators

$$[\rho_{p,n}, \rho_{p',n'}] = -p\frac{n\nu}{L_p^2}\delta_{n,-n'}\delta_{p,p'}, \quad \text{(B13)}$$

and for the Fourier components of the phase operators,

$$[\phi_{p,n}, \phi_{p',n'}] = -p\frac{\nu}{n}\delta_{n,-n'}\delta_{p,p'}. \quad \text{(B14)}$$

Note that the Fourier components of field operators from opposite edges $p = \pm$ do commute. This is a consequence of our choice of the basis as the set of Eigen modes on each separate edge.

quantization of the 0- modes:

Special care has to be taken of the zero modes, $n = 0$. While the nonzero modes $\rho_{p,n}$ describe fluctuations of the edge which do not change the total charge on each edge, and which have a linear dispersion, N_p is the total number operator of quasiparticles with charge $q = \nu e$ on edge $p = \pm$ and its conjugate ϕ_{p0} is related to the ladder operator, adding a particle to the edge p, as $\exp(i/\nu\phi_{p0})$ We do obtain the algebra of the quantized 0- modes by demanding that

1. the ladder operator indeed adds a quasiparticle to edge p:

$$\exp(i\phi_{p0})N_p = (N_p + 1)\exp(i\phi_{p0}). \quad \text{(B15)}$$

2. The bosonized Fermion operators obey Fermi statistics, and thus anticommute:

$$\{\psi_p(x), \psi_{p'}(x')\} = 0. \quad \text{(B16)}$$

Anticommutation between Fermi operators on the same edge:

With $\psi_p^{1/\nu}(x) = C_p^{1/\nu}\exp(i(1/\nu)\phi_p(x))$ follows, with the Baker- Haussdorff formula for equal edges, $p = p'$:

$$(\exp(-\frac{1}{2\nu^2}[\phi_p(x), \phi_p(x')]) + \exp(\frac{1}{2\nu^2}[\phi_p(x), \phi_p(x')]))$$
$$C_p^{2/\nu}\exp(i\frac{1}{\nu}(\phi_p(x) + \phi_p(x')) = 0, \quad \text{(B17)}$$

which means that

$$[\phi_p(x), \phi_p(x')] = -ip\nu^2(2s+1)\pi sgn(x-x'). \quad \text{(B18)}$$

Thus, the condition of anticommutation of the bosonized fermion operators still leaves the freedom to choose the integer s. This freedom, however is lost when we demand that

1 . $\psi_p^{1/\nu}(x)$ changes the density $\rho_p(x')$ by $-1/L\delta(x-x')$, corresponding to a removal of a charge of an electron and

2. the phase accumulated under the exchange of two quasiparticles at the edge is $\pi\nu$ as in the bulk of the FQH liquid. Thus, we obtain the condition $2s + 1 = 1/\nu$.

Note that the phase accumulated under exchange of two edge fermions is then $1/\nu\pi$ rather than π as for usual fermions.

Now, we will use these commutation relations in real space to find the commutation relations of the zero modes, which is possible, since we do know already the commutation relations of the nonzero modes which we had derived above from the Hamiltonian equations combined with the continuity equation.

For $p = p'$, it follows from Eqs. (B4,B13,B18) that

$$[N_p, \phi_{p0}] = i. \quad \text{(B19)}$$

anticommutation between fermi operators defined on opposite edges:

Next, demanding the anticommutation of fermion operators defined on opposite edges $p \neq p'$, it follows

$$C_p = \exp(i\nu^2(2s+1)p\frac{\pi}{2}N_{-p}). \quad \text{(B20)}$$

Demanding additionally that the exchange of two quasiparticles on different edges gives a phase $\pm\nu\pi$, the freedom in s is lifted: $2s + 1 = 1/\nu$.

Algebra of ϕ and θ:

Having now the complete set of commutation relations in real and in momentum space we now look for conjugate operators which are not self dual in real space, which means that we look for operators which do commute taken at different points in space.

One finds, using Eq. (B18), that these nonself dual operators are the symmetric and antisymmetric combinations of the operators on opposite edges:

$$\phi(x) = \phi_1(x) + \phi_2(x), \quad \text{(B21)}$$

and

$$\theta(x) = \phi_1(x) - \phi_2(x). \tag{B22}$$

We find that ϕ and θ do not commute with each other:

$$[\phi(x), \theta(x')] = -4\pi\nu i\theta_{step}(x - x'). \tag{B23}$$

However, each is not self dual:

$$[\phi(x), \phi(x')] = 0, \tag{B24}$$

and

$$[\theta(x), \theta(x')] = 0. \tag{B25}$$

APPENDIX C: CHIRAL ACTION

Here we give the path integral derivation of the functional integral representation of the partition function of a chiral edge state of an incompressible FQH liquid. This turns out to be nontrivial due to the chirality of the edge states, the selfduality of the chiral edge field operators, and the dynamics of the 0- modes in a finite FQH- annulus. We sketch this derivation for the case of the FQH-annulus in the following.

The partition function is given by

$$Z = Tr[e^{-\beta\hat{H}}] \tag{C1}$$

Dividing the inverse temperature β into infinitesimally small imaginary time slices ϵ with $M\epsilon = \beta$ one obtains

$$\begin{aligned} Z &= Tr \prod_{j=0}^{M} e^{-\epsilon\hat{H}} \\ &= Tr \prod_{j=0}^{M} : e^{-\epsilon\hat{H}} : . \end{aligned} \tag{C2}$$

Here : ... : denotes normal ordering, and $\hat{H}$ is assumed to be written in terms of conjugate operators $\hat{\phi}, \hat{\phi}^*$. The correction to the normal ordered factors are of order ϵ^2 each and can therefore be neglected in the limit $\epsilon \to 0$, $M \to \infty$, while $M\epsilon = \beta$ is kept fixed to the inverse temperature β. [18]

For the chiral edge, there arises the problem in the derivation of the functional integral, that the operators defined on one chiral edge are selfdual, see appendix B, so that one can not straightforwardly apply the Feynman path integral method. Therefore, one has to find nonselfdual combinations of these operators and their conjugates. One possible choice are the combinations ϕ and θ of the fields defined on opposite edges, Eqs. (B21,B22). However, we find it more convenient to divide each chiral edge into two parts, and form nonselfdual operators as combinations of operators defined in these two parts. We first sketch the derivation for a single chiral edge of length L_p, and can then easily extend the result to several edges.

The combinations $\hat{\phi}_{s,a} = \hat{\phi}_p(x) \pm \hat{\phi}_p(-x)$ of the chiral fields $\hat{\phi}_p(x)$ are not selfdual for $0 \le x \le L_p/2$:

$$[\hat{\phi}_{s,a}(x), \hat{\phi}_{s,a}(x')] = 0. \tag{C3}$$

while they do not commute with each other,

$$[\hat{\phi}_s(x), \hat{\phi}_a(x')] = p4\pi i\nu\theta_{step}(x' - x). \tag{C4}$$

Now, we can continue in the derivation of the functional integral in terms of the Eigen values of the conjugate opertaors

$$\hat{\phi}(x) = \hat{\phi}_s(x), \tag{C5}$$

$$\hat{\phi}^*(x) = \frac{1}{4\pi i\nu^2(2s+1)}\partial_x\hat{\phi}_a(x). \tag{C6}$$

with

$$[\hat{\phi}(x), \hat{\phi}^*(x')] = \delta(x - x') \tag{C7}$$

The respective coherent states are

$$\mid \phi >= e^{\int_0^{L_p/2} dx\phi(x)\hat{\phi}^*(x)} \mid 0 >, \tag{C8}$$

which satisfy the completeness relation,

$$\int \prod_{x=0}^{L_p} \frac{d\phi^*(x)d\phi(x)}{2\pi i} e^{-\int_0^{L_p/2} dx\phi^*(x)\phi(x)} \mid \phi >< \phi \mid = 1. \tag{C9}$$

Then, the partition function is obtained as

$$\begin{aligned} Z = \int \prod_{j=0}^{M} \prod_{x=0}^{L_p/2} \frac{d\phi^{(j)*}(x)d\phi^{(j)}(x)}{2\pi i} e^{-\sum_{j=0}^{M}\int_0^{L_p/2} dx\phi^{(j)*}(x)\phi^{(j)}(x)} \\ \prod_{j=0}^{M} < \phi^{(j+1)} \mid : e^{-\epsilon\hat{H}[\hat{\phi},\hat{\phi}^*]} : \mid \phi^{(j)} > . \end{aligned} \tag{C10}$$

Because of the normal ordering, the matrix elements in coherent state representation at each time slice are easily evaluated as

$$\begin{aligned} &< \phi^{(j+1)} \mid : e^{-\epsilon\hat{H}[\hat{\phi},\hat{\phi}^*]} : \mid \phi^{(j)} > \\ &= < \phi^{(j+1)} \mid \phi^{(j)} > e^{-\epsilon H[\phi^{(j)},\phi^{(j+1)*}]}. \end{aligned} \tag{C11}$$

Having thus the functional integral representation one can do a transformation back to the fields defined on a single point, $\phi_p(x)$, yielding the correct effective action of one chiral edge, which can be checked by variation of this effective action, giving the correct equations of motion,

$$S = S_0 - \int_0^\beta d\tau p \frac{i}{4\pi\nu} \phi_p(x) \partial_\tau \phi_p(-x) \mid_0^{L_p/2}, \quad \text{(C12)}$$

where $\phi_p(x)$ is the chiral field defined on one edge. Here, S_0 is the action one would obtain, disregarding the self-dual nature of the operators on a chiral edge:

$$S_0 = \int_0^\beta d\tau \left(\int_{-L_p/2}^{L_p/2} dxp \frac{i}{4\pi\nu} (\partial_\tau \partial_x \phi_p(x)) \phi_p(-x) + H[\phi_p(x)] \right). \quad \text{(C13)}$$

Disregarding the dynamics of the 0 - modes, one would indeed obtain $S = S_0$. However, including the dynamics of the 0- modes, which are the particle number N_p and its phase conjugate ϕ_{p0}, S_0 itself contains a term which couples the 0- modes to the nonzero- modes, giving an unphysical term to the equations of motions, as obtained by variation of the action S_0. The additional term appearing in Eq. (C12) cancels exactly this unphysical term contained in S_0, thus yielding the correct equations of motion.

Having the partition function of a single chiral edge, we can easily give the one for a FQH- annulus with two edges: Noting that the Hamiltonian can now be written as a functional of $\hat{\phi}_p$, $p = \pm$, and that $\hat{\phi}_+(x)$ and $\hat{\phi}_-(x)$ do commute with each other, we obtain for the total action of the FQH- annulus

$$S = S_0 - \int_0^\beta d\tau \sum_{p=\pm} p \frac{i}{4\pi\nu} \phi_p(x) \partial_\tau \phi_p(-x) \mid_0^{L_p/2}, \quad \text{(C14)}$$

where

$$S_0 = \int_0^\beta d\tau \Big(\sum_{p=\pm} p \frac{i}{4\pi\nu} \int_{-L_p/2}^{L_p/2} dx \partial_\tau \partial_x \phi_p(x)) \phi_p(x) + H[\phi_+(x), \phi_-(x)] \Big) \quad \text{(C15)}$$

APPENDIX D: RENORMALIZATION

Here we give a derivation in perturbation theory of the Renormalization flow equations for the backscattering amplitude.

Unlike the route of derivation in real space taken by Kane and Fisher [6] we find it more convenient to give the derivation in momentum space.

Since the scattering term is local in real space, the matrix in the Gaussian integral is nondiagonal in momentum space. However, we show in the following that to first order in the tunneling amplitudes t_m, only the diagonal elements of this matrix contribute to the renormalization. Consider a Gaussian integral of the form

$$\int \prod_i dz_i dz_i^* \exp(-\sum_{ij} z_i A_{ij} z_j^*) = 1/detA = \exp(-Tr \ln A) \quad \text{(D1)}$$

When A has the form $A = A_0 + tB$ where t is a small parameter, one can expand in t and find to first order in t:

$$\exp(-Tr \ln A_0 - Trt \frac{B}{A_0}) \quad \text{(D2)}$$

When A_0 is a diagonal matrix, we find that, indeed t is renormalized by the diagonal matrix elements of B, only, which we wanted to prove:

$$\exp(-\sum_i (\ln A_{0ii} - t \frac{B_{ii}}{A_{0ii}})) \quad \text{(D3)}$$

We will use this result now in the derivation of the renormalization group equation of the backscattering amplitude t_m.

Before giving the explicit calculation, let us sketch the RG analysis of the back scattering amplitude which we want to perform in order to find its effective dependence on the relevant energy scale of the system $E = max\{T, v/R\}$, temperature or level spacing. To this end we go to Matsubara frequency representation and integrate out all Fourier components of all the fields in the functional integral in an infinitesimal energy window $\Lambda/b < \omega_s < \Lambda$ where $b = 1 + dl$. After that, we have to rescale the size of the system by transforming $\beta \to b\beta$ which also transforms the Matsubara frequencies as $\omega_l \to \omega_l/b$. Then, one has to rescale the remaining field variables such that the effective back scattering term in the action has again the original form before integration over the fields in the energy window, but now with a renormalizad back scattering amplitude. Then, we repeat this renormalization step M times until we reach the physically relevant energy scale E of the system, so that M is given by $\Lambda/b^M = E$.

The partition function of the FQH- annulus with a weak impurity at position $x = 0$ is given as a functional integral in the momentum and Matsubara frequency representation:

$$Z = \int \prod_{\omega_s} dJ(\omega_s) d\phi_J(\omega_s) \prod_{n\neq 0} d\phi_{-,n}(\omega_s) d\phi_{+,n}(\omega_s)) \exp(-S) \quad \text{(D4)}$$

where $\omega_s = 2\pi T s$ are the bosonic Matsubara frequencies. The action S is for the FQH- annulus with a weak impurity given by

$$S = S_0 + S_{BS}, \quad \text{(D5)}$$

where S_0 is the action without back scattering between the edges,

$$S_0 = T \sum_{\omega_s} (-\frac{1}{2} \omega_s \phi_J(\omega_s) J(-\omega_s) + \frac{\nu v}{4R} J(\omega_s) J(-\omega_s)$$

$$+\sum_{n>0}\sum_{p=\pm}(-\frac{n}{\nu}i\omega_s+\frac{v_p}{\nu R_p}n^2)\phi_{p,n}(\omega_s)\phi_{p,-n}(-\omega_s))$$
$$+\frac{\nu v}{R}(-\varphi J(\omega_s=0)+\beta\varphi^2). \quad \text{(D6)}$$

where $v/R=\sum_p v_p/R_p/2$. We consider the part of the action which describes back scattering by taking into account only Gaussian fluctuations about its minima, that's when cos gives -1. Thus, we expand, $\cos x = -1+1/2(x-(2r+1)\pi)^2$, and include the sum over the different minima r, in the partition function. This Gaussian approximation of the back scatttering action thus gives in Matsubara frequency representation a term

$$S_{BS}=\sum_{m=1}^{\infty}t_m(-\beta(1-\frac{1}{2}(2r+1)^2\pi^2)$$
$$-(2r+1)\pi m(\phi_J(\omega_s=0)+\sum_{n\neq 0}\sum_{p=\pm}p\phi_p(\omega_s=0))$$
$$+\frac{1}{2}m^2T\sum_{\omega_s}(\phi_J(\omega_s)+\sum_n\sum_{p=\pm}p\phi_{p,n}(\omega_s))$$
$$(\phi_J(-\omega_s)+\sum_{n'}\sum_{p=\pm}p\phi_{p,n'}(-\omega_s))). \quad \text{(D7)}$$

Note that while S_0 is diagonal both in Matsubara frequency and momentum space, the back scattering term S_{BS} is nondiagonal in momentum space. However, as we have seen above, only the diagonal matrix elements of S_{BS} contribute to the renormalization of the back scattering amplitudes t_m, so that we can easily proceed in the renormalization procedure by integrating out those fileds $J(\omega_s),\phi_J(\omega_s)$ $\phi_{p,n}(\omega_s)$ with energy in the interval $\Lambda/b<\mid\omega_s\mid<\Lambda$. As a result, we obtain to first order in t_m the following additional terms:

$$\sum_{\Lambda/b<|\omega_s|<\Lambda}\sum_{m=1}^{\infty}\frac{1}{2}t_m m^2(\sum_p\sum_{n\neq 0}\frac{1}{A_{p,n}(\omega_s)}+\frac{1}{A_0(\omega_s)}), \quad \text{(D8)}$$

where $A_{p,n}(\omega_s)=n/\nu(-i\omega_s+2\pi v/Ln)$ and $A_0(\omega_s)=-R/(2\nu v)\omega_s^2$. Performing the summations over momentum and Matsubara frequency, we obtain, noting that $\Lambda\gg v/L$ that the back scattering amplitudes t_m become renormalized by

$$\beta\tilde{t}_m=\beta t_m(1-\nu m^2\ln b-\frac{\nu m^2 v}{2\pi R\Lambda}(b-1)) \quad \text{(D9)}$$

Before repeating this procedure of integrating over an infinitesimal energy window, we have to bring the action to the original form it had before the integration. To this end we have to blow the shrinked size of the system up by $\beta\to b\beta$ and accordingly substitute for the Matsubara frequencies $\omega_s\to\omega_s/b$. Finally one has to transform the fields $\phi_{p,n}(\omega_s)\to C_1\phi_{p,n}(\omega_s)$, where the factor $C_1=\sqrt{\sum_{m=1}^{\infty}t_m}/\sqrt{\sum_{m=1}^{\infty}\tilde{t}_m}$ ensures that the back scattering action is transformed to the form it had before the integration, in Gaussian approximation. Now we start again by integrating over the next infinitesimal energy window. We have to keep on repeating this renormalization procedure, until the upper cutoff of the action is equal to a typical energy E of the system: $\Lambda/b^M=E$. Here, M is the number of renormalization cycles. Thus, we obtain that the back scattering amplitude flows as a function of the typical energy E like

$$t_m\to t_m b^{M-1}\prod_{j=1}^{M}((1-m^2\nu\ln b-\frac{\nu m^2 v}{L}\frac{b^{j-1}}{\Lambda}(b-1))$$
$$\approx t_m b^{M(1-m^2\nu)}e^{-\nu m^2 v/(L\Lambda)b^M}. \quad \text{(D10)}$$

which gives with $b^M=\Lambda/E$:

$$t_m\to t_m(E/\Lambda)^{m^2\nu-1}e^{-\nu m^2 v/L/E}. \quad \text{(D11)}$$

Thus, for the system with a finite level spacing v/R, the relevant power law renormalization found in Ref. [6], is modified at temperatures close to the level spacing by a factor which depresses the tunneling amplitude. This behavior is well known as the Coulomb blockade effect, where here the charging energy is the level spacing v/R, the energy of an additional particle on an edge. As the temperature is lowered below the level spacing, however, the typical energy scale E is the level spacing v/R, and the renormalized tunneling amplitude converges to a temperature independent value.

APPENDIX E: SERIES AND INTEGRALS

Here, we perform the summation over n in the expression

$$D(\tau)+D(-\tau)=\nu\sum_p\sum_{n>0}\frac{1}{n}\frac{\cosh(n\pi v_p/(L_pT)(1-\mid\tau\mid))}{\sinh(n\pi v_p/(L_pT))}). \quad \text{(E1)}$$

We assume here that the level spacings on each edge are of similar magnitude. Then, we can treat two different temperature regimes:

For small temperatures below the level spacing v/R, $v/(RT)\gg 1$, one can approximate $\cosh(n\pi v/(TL)(1-T\mid\tau\mid))/\sinh(n\pi v/(L))\sim\exp(-n\pi v/L\mid\tau\mid)$, and the summation over n can be performed, yielding:

$$D(\tau)+D(-\tau)=-2\nu\ln(1-e^{-\pi v/L|\tau|}). \quad \text{(E2)}$$

Denoting the ultraviolet cutoff by Λ one obtains then also

$$D(0)=-\nu\ln(\frac{v}{2R\Lambda}), \quad \text{(E3)}$$

In the other limit, when the temperature exceeds the level spacing, $v/(RT) \ll 1$, the summation can be transformed into a continous integral over $x = vn/T/L$ with a lower cutoff $\epsilon \ll 1$ and one obtains:

$$D(\tau) + D(-\tau) = 2\nu \int_0^\infty dx \frac{x}{x^2+\epsilon^2} \frac{\cosh(\pi(1 - T \mid \tau \mid)x)}{\sinh(\pi x)}. \tag{E4}$$

The integral can be performed (Gradstein/ Ryshik 4.115.11., and 1.441.4. [19]) and we obtain with $\epsilon \approx v/T/L$:

$$D(\tau) + D(-\tau) = \nu TL/v - 2\nu \ln[2\sin(\pi/2T \mid \tau \mid)]. \tag{E5}$$

and with the ultraviolet cutoff Λ,

$$D(0) = \nu/2TL/v - \nu \ln[\pi T/\Lambda]. \tag{E6}$$

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Figures

Fig.2 : The persistent current in a FQH- annulus with weak constriction, at temperature to level spacing ratios, $t/(v/R) = .1, .05, .01$.

Fig. 3 : The persistent current in a clean FQH-annulus at $T/(v/R) = .1, .05, .01$.